Problem Set 2

Solution

1. [50/100]

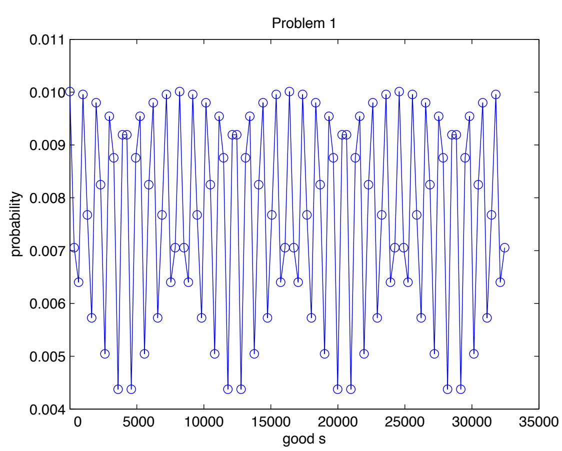
Solution:

We define a value s is "good" if

$$\big(0 \leq sr \mod M \leq \frac{r}{2}\big) \;\; or \;\; \big(M - \frac{r}{2} \leq sr \mod M \leq M-1\big).$$

The probability of each "good s" is give by

$$\left\|\frac{1}{\sqrt{\lceil \frac{M}{r} \rceil}} \frac{1}{\sqrt{M}} \sum_{t=0}^{\lceil \frac{M}{r} \rceil - 1} w^{(x_0+tr)s}\right\|^2$$



2. [50/100]

Solution:

Note that

$$\sum_{x \in \{0,1\}^n} f(x)|x\rangle = \frac{1}{2^{n-k}} \sum_{z \in \{0,1\}^{n-k}} |0\rangle^{\otimes k}|z\rangle.$$

Applying the Hadamard transform,

$$\begin{aligned}
H^{\otimes n}\left[\sum_{x \in \{0,1\}^n} f(x)|x\rangle\right] &= H^{\otimes n}\left[\frac{1}{2^{n-k}} \sum_{z \in \{0,1\}^{n-k}} |0\rangle^{\otimes k}|z\rangle\right] \\
&= \left[H^{\otimes k}|0\rangle^{\otimes k}\right] \otimes H^{\otimes n-k}\left[\frac{1}{2^{n-k}} \sum_{z \in \{0,1\}^{n-k}} |z\rangle\right] \\
&= \left(\frac{1}{\sqrt{2}}|0\rangle + |1\rangle\right)^{\otimes k} \otimes |0\rangle^{\otimes n-k} \\
&= \frac{1}{\sqrt{2^k}} \sum_{y \in \{0,1\}^k} |y\rangle|0\rangle^{\otimes n-k} \\
&= \sum_s \hat{f}(s)|s\rangle =: q
\end{aligned}$$

Thus, $\hat{f}(s)$ can be found as

$$\hat{f}(s) = \begin{cases} \frac{1}{\sqrt{2^k}} & \text{if last } n-k \text{ bits of } s \text{ are all zeros} \\ 0 & \text{otherwise} \end{cases}$$

Hence when we measure q, we attain the uniform distribution over the elements $\{0,1\}^n$, of which the last $n-k$ bits are zeros, and the probability of any of such s is given by

$$\left|\hat{f}(s)\right|^2 = \frac{1}{2^k}$$