Problem Set 3

This version has some corrections in problem 1, posted October 23, 2012

This problem set is due on Thursday, October 25, by 2:15pm. You can either hand it in class or email a pdf to Joongyeub.

1. [50/100] At the end of the third step we have the quantum state

$$\frac{1}{\sqrt{\lceil \frac{M}{r} \rceil}} \frac{1}{\sqrt{M}} \sum_{s=0}^{M-1} \omega^{x_0 s} \sum_{t=0}^{\lceil \frac{M}{r} \rceil -1} \omega^{trs} |s\rangle$$

   where $f(x_0)$ is the value measured at the second step and $\omega = e^{-2\pi i/M}$, and recall that we defined a value $s \in \{0, \ldots, M-1\}$ to be "good" if

$$\left(0 \leq sr \bmod M \leq \frac{r}{2}\right) \text{ or } \left(M - \frac{r}{2} \leq sr \bmod M \leq M-1\right)$$

   Suppose that we are running the period-finding algorithm with $M = 2^{15}$ and $r = 100$, and that at the second step we measured $f(x_0)$ where $x_0 = 3$. Using the computational mathematics package or the programming language of your choice, find all the good $s \in \{0, \ldots, 2^{15} - 1\}$ and compute the absolute value of the amplitude-squared (i.e. the probability of being measured) of each good s. Print out the list of good s and their probability and the code you used to generate them.

2. [50/100] It follows from the analysis of the period-finding algorithm that if $\Omega := \{0, \ldots, M-1\}$, r is a divisor of M, and

$$\sum_{x \in \Omega} f(x)|x\rangle$$

   is a uniform superposition of the x that are multiples of r (that is, $f(x) = \sqrt{r/M}$ if x is a multiple of r, and $f(x) = 0$ otherwise), then measuring the quantum state

$$\sum_{s \in \Omega} \hat{f}(s)|s\rangle$$

gives us the uniform distribution over s that are multiples of M/r (that is, $|\hat{f}(s)|^2 = 1/r$ is s is multiple of M/r and $\hat{f}(s) = 0$ otherwise). on-zero precisely on the multiples of M/r.

In this problem we prove a "Boolean version" of the above fact. Let $\Omega := \{0,1\}^n$ and suppose that

$$\sum_{x \in \{0,1\}^n} f(x)|x\rangle$$

is a quantum state that is a uniform superposition of the strings x that have zeroes in the first k coordinates (that is $f(x) = 1/\sqrt{2^{n-k}}$ if the first k bits of x are zero, and $f(x) = 0$ otherwise), and consider the quantum state

$$q := \sum_{s \in \{0,1\}^n} \hat{f}(s)|s\rangle$$

where $\hat{f}$ is the Hadamard transform of f. Compute the amplitudes $\hat{f}(s)$ for every s, and describe the probability distribution of outcomes that we get by measuring q.