Problem 1 Solution

$$\left((01) \cup \left((1 \cup 00)(10)^*(0 \cup 11)\right)\right)^* (1 \cup 00)(10)^*$$

Problem 2 Solution

Let $a = (00 \cup 01 \cup 10)$.

The answer is $\left(\left((11)a^*(11)\right) \cup a\right)^* (11)a^*$

Problem 3 Solution

$(0 \cup 1)^+ 1(0 \cup 1)^3$

Problem 4 Solution

A regular expression can be defined as follows (where M represents Σ)

```
Regex = Symbol M
      | Concat Regex Regex
      | Choice Regex Regex
      | Star Regex
      | Null
```

this states that a regular expression is one of:

- a symbol of Σ

- concatenation of two regular expressions

- choice of two regular expressions

- or star of a regular expression

Now, consider the following function:

```
reverse :: Regex -> Regex
reverse (Symbol x) = Symbol x
reverse (Concat regA regB) = Concat (reverse regB) (reverse regA)
reverse (Choice regA regB) = Choice (reverse regA) (reverse regB)
reverse (Star regA) = Star (reverse regA)
reverse (Null) = Null
```

We claim that for all regular expressions r, reverse(r) accepts the reverse of the language accepted by r. The proof of this follows by structural induction on regular expressions.