Pactice Midterm 2

Problem 1

Show that the language $L = \{\langle M \rangle :$ there exists a polynomial $p(\cdot)$ such that, for every $x \in \{0,1\}^*$, $M(x)$ halts in time $\leq p(|x|)\}$ is undecidable.

Problem 2

Define $L = \{\langle M_1, M_2 \rangle : L(M_1) = \overline{L(M_2)}\}$.

Show that L is not recognizable.

Problem 3

Two binary strings x and y are ε-close if $|x| = |y|$, and x and y differ in at most $\varepsilon \cdot |x|$ positions.

Prove that $Q(x)$ =min$\{K(y)|x, y$ are .0001-close$\}$ is not computable. You can use the fact that $\binom{n}{k} \leq \left(\frac{e \cdot n}{k}\right)^k$.

Problem 4

Show that for every constant c, there are strings x, y such that $K(xy) > K(x) + K(y) + c$.

(Note: Don't get hung up on this problem. It is quite hard and is meant as a challenge if the other problems are completed)

Problem 5

Consider the language consisting of (G, K) pairs, where G is a **directed** graph that can be made acyclic by removing k edges. Formally, the feedback arc set problem is $L = \{\langle G = (V, E), k\rangle : \exists S \subseteq E, |S| = k, G - S \text{ is acyclic}\}$.

Prove that L is NP-complete.

Problem 6

Consider a crossword-puzzle game to be:

An $m \times n$ matrix (the board), where each entry is

"*" (blank, can fill in a letter) or

"#" (can't fill in a letter)

This can give us our jagged crossword board we're used to where we have to fill in the blanks.

Furthermore, we have an alphabet. Say, $\Sigma = \{a, b, c\}$.

And finally we have a language $L \subset \Sigma^*$ be some finite list of words.

The crossword problem is then: given a board, can we fill in each "*" with a letter from the alphabet such that

- Every column (read from top to bottom) is a sequence of words from L, separated by one or more #s
- Every row (read from left to right) is a sequence of words from L, separated by one or more #s

Show that the crossword problem is NP-complete.

[Hint: there is more than one way to prove this result. One possibility is to work from Vertex Cover; note that you don't even need to use the #s]
