Pactice Final

Problem 1

Prove there exists an infinite sequence of languages $L_1, L_2, ...$ such that:

- For every i, L_i is neither recognizable nor co-recognizable
- $L = \cap_i L_i$ is regular
- $|L| = \infty$

Problem 2

Prove: $NEXP \neq EXP \Rightarrow NP \neq P$

Problem 3

Prove: $P = NP$ iff there exists $k, l > 2$ such that $NTime(O(n^k)) \subseteq DTime(O(n^l))$

Problem 4

For $f : \{0,1\}^n \to \{0,1\}$, let $size(f)$ be the size of the smallest circuit which implements function f.

Prove: forall $T(n) \leq 2^n/1000n$, there exists $f : \{0,1\}^n \to \{0,1\}$ such that $T(n) \leq size(f) \leq T(n) + 10n$.

You can use the fact that for every n, there are functions $f : \{0,1\}^n \to \{0,1\}$ that can not be computed by circuits of size $< 2^n/1000n$.

Problem 5

Let $K(x)$ be the Kolmogorov complexity of x.
Let X_n be the set of all TMs M where (1) $|Q| + |\Gamma| \leq n$ and (2) M halts on the empty input.
Let $BB(n)$ be the longest time any TM in X_n runs on the empty input.

Design an algorithm A such that $A^{BB}(x) = K(x)$. (A is granted a blackbox routine that computes BB, and is allowed to query it.)

Problem 6

An Eulerian cycle of an undirected graph is a cycle that visits every edge exactly once.

For every integer n, consider the streaming complexity of the problem of deciding whether a graph on n vertices, given by a stream of edges, has an Eulerian cycle.

That is, for a set n vertices V, our alphabet $\Sigma = \{\{v1, v2\} \mid v1, v2 \in V, v1 \neq v2\}$ is all possible (undirected) edges between these vertices and our stream is a sequence of these edges. If we call the set of each edge in this stream E, then $G = (V, E)$ is the undirected graph defined by it. We want a streaming algorithm that takes the stream and computes whether or not G has an Eulerian cycle. Show that the bits of memory required for a streaming algorithm for this problem is $\Omega(n)$.

Problem 7

Undirected graphs G and H are isomorphic iff there exist a bijection $f : G_V \to H_V$ such that $(u, v) \in G_E$ iff $(f(u), f(v)) \in H_E$.

Let $L = \{\langle G, H \rangle | (G, H) \text{ are isomorphic } \}$.

Define a zero-knowledge protocol for L.

Prove that it is complete, sound, and perfect zero-knowledge.

Hint: Prover sends a permuted graph. Verifier sends a bit. Prover sends a permutation.