Problem Set 9

This problem set is due on Friday, April 17, by 5pm. Please submit your solution online using bcourses, as a pdf file. You can type your solution, or handwrite it. If you handwrite it, then either scan it or take a good resolution picture of each page and then collate the pictures and export them to a *single* pdf file.

When asked to prove that a problem is NL-complete, make sure to include a proof that the problem is in NL. When specifying an NL algorithm, preferably give pseudo-code for it, and use terminology like "guess $x \in S$" to specify a non-deterministic step in which variable x can take any of the values in the set S.

Problem 1 (35/100)

Consider the *acceptance problem for NFA*, that is the language

$$A_{NFA} := \{(M, w) : M \text{ is an NFA } \wedge M \text{ accepts } w\}$$

Show that A_{NFA} is NL-complete under $\leq_m^{\log}$ reductions.
[You can use the fact that the reachability problem for directed graphs is NL-complete]

Problem 2 (35/100)

Consider the DAG-Reach problem defined as follows

$$DAG - Reach := \{(G, s, t) : G \text{ G is a DAG} \wedge t \text{ is reachable from } s\}$$

(where DAG stands for directed acyclic graph). Show that DAG-Reach is NL-complete. (This shows that the reachability problem in directed graphs remains NL-complete even when restricted to acyclic graphs.)
[Hint: create multiple copies of each vertex/configuration, and arrange them in layers.]

Problem 3 (30/100)

Consider the language

$$StrongConn := \{G : G \text{ is a strongly connected directed graph}\}$$

Recall that a graph $G = (V, E)$ is strongly connected if for every two vertices u, v there is a part from u to v and also a path from v to u. Prove that StrongConn is NL-complete.
[Hint: reduce from DAG-Reach]