

Problem Set 8

This problem set is due on Friday, April 3rd, by 5pm. Please submit your solution online using bcourses, as a pdf file. You can type your solution, or handwrite it. If you handwrite it, then either scan it or take a good resolution picture of each page and then collate the pictures and export them to a *single* pdf file.

Problem 1: NFA Minimization (30)

In the NFA minimization problem, we are given a NFA N_1 and want to output a NFA N_2 that accepts the same language using as few states as possible.

Prove that if there is a polynomial time for NFA minimization, then $P = NP$.

Problem 2: ExactClique is both NP-hard and coNP-hard (30)

Let $ExactClique = \{\langle G, k \rangle \mid \text{the largest clique in } G \text{ is of size exactly } k\}$.

Prove that $ExactClique$ is both NP-hard and coNP-hard.

Problem 3: Circuit Minimization (40)

In the Circuit Minimization problem (CircuitMin), we are given a boolean circuit C_1 , and want to output a boolean circuit C_2 that accepts the same language using as few gates as possible. In situations where multiple circuits of minimal size exists, outputting any circuit of minimal size is acceptable. Assume the only available gates are: **and** :: $\{0, 1\} \times \{0, 1\} \rightarrow \{0, 1\}$, **or** :: $\{0, 1\} \times \{0, 1\} \rightarrow \{0, 1\}$, **not** :: $\{0, 1\} \rightarrow \{0, 1\}$.

Prove that $P = NP$ if and only if CircuitMin can be solved in polynomial time.