Problem Set 6

This problem set is due on Friday, March 13, by 5pm. Please submit your solution online using bcourses, as a pdf file.

You can type your solution, or handwrite it. If you handwrite it, then either scan it or take a good resolution picture of each page and then collate the pictures and export them to a *single* pdf file.

Problem 1: Unary Decidability (30)

Let $\Sigma = \{1\}$. Prove that there is an undecidable subset of Σ^*.

Problem 2: Recognizable-Decidable (30)

Prove that a language L is recognizable iff there exists a decidable language D such that $x \in L \Leftrightarrow \exists y : (x, y) \in D$.

Problem 3: Reductions Revisited (40)

Let A be a language that is neither empty nor Σ^*. Consider the two sets of languages:

- $L = \{B \mid B \leq_m A\}$
- $U = \{B \mid A \leq_m B\}$

Prove that L is countable and U is uncountable.