Problem Set 5

This problem set is due on Friday, March 6, by 5pm. Please submit your solution online using bcourses, as a pdf file.

You can type your solution, or handwrite it. If you handwrite it, then either scan it or take a good resolution picture of each page and then collate the pictures and export them to a *single* pdf file.

Instructions

Please follow the provided format. The grading rubric is written based on the provided format.

You may use without proof that neither $\overline{A_{TM}}$ nor $\overline{H_{TM}}$ is recognizable where $A_{TM} = \{\langle M, x\rangle \mid M(x) \text{ accepts}\}$ and $H_{TM} = \{\langle M, x\rangle \mid M(x) \text{ halts}\}$.

Problem 1: Regex Decidability (30)

Let $A = \{\langle R, S\rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$. Construct an algorithm that decides A. You need to prove that your algorithm (a) is correct and (b) always terminates.

Hint: describe and analyze an algorithm that, given R and S, constructs a DFA $M_{R,S}$ such that $L(R) \subseteq L(S)$ if and only if $M_{R,S}$ rejects every input. You can then use (withouth proof) the fact there is a polynomial time algorithm that, given a DFA M, decides whether M rejects every input.

Problem 2: TM Recognizability (30)

Let $A = \{\langle R, S\rangle \mid R \text{ and } S \text{ are TMs and } L(R) \subseteq L(S)\}$. In this multipart problem, we will prove that neither A nor $\overline{A}$ is Turing-recognizable.

(a) (15 points) Show that given a recognizer for A, you can devise a recognizer for $\overline{A_{TM}}$.

(b) (15 points) Show that given a recognizer for $\overline{A}$, you can devise a recognizer for $\overline{A_{TM}}$.

Problem 3: Bounded Recognizability (40)

Let $L = \{\langle M\rangle \mid \text{ for all strings } x, M(x) \text{ halts within } |x|^2 \text{ steps }\}$, where $|x|$ denotes the length of a string x. In this multipart problem, we will prove that L is not recognizable, but $\overline{L}$ is recognizable.

(a) (10 points) Design a recognizer for $\overline{L}$.

(b) (30 points) Show that given a recognizer for L, you can devise a recognizer for $\overline{H_{TM}}$.