Problem Set 3

This problem set is due on Friday, February 13, by 5pm. Please submit your solution online using bcourses, as a pdf file. You can type your solution, or handwrite it. If you handwrite it, then either scan it or take a good resolution picture of each page and then collate the pictures and export them to a *single* pdf file.

Problem 1: Minimal Automata (10/100)

Consider the language $L = \{x \in \{0,1\}^* :$ the number of 1's in x is a multiple of $3\}$. Draw a minimal DFA with states labeled with the equivalence classes $[\varepsilon]$, [1], and [11] defined by indistinguishability over L.

Problem 2: Regular? (20/100)

Are the following languages regular? If so, draw a minimal DFA for it with each state labeled with its corresponding equivalence class (signified by a representative for that class under the indistinguishability relation). If not, prove this with the Myhill-Nerode theorem by exhibiting infinitely many distinguishable strings.

a) $L = \{x \in \{a,b\}^* :$ the number of b's in x has a remainder of 3 when divided by $5\}$

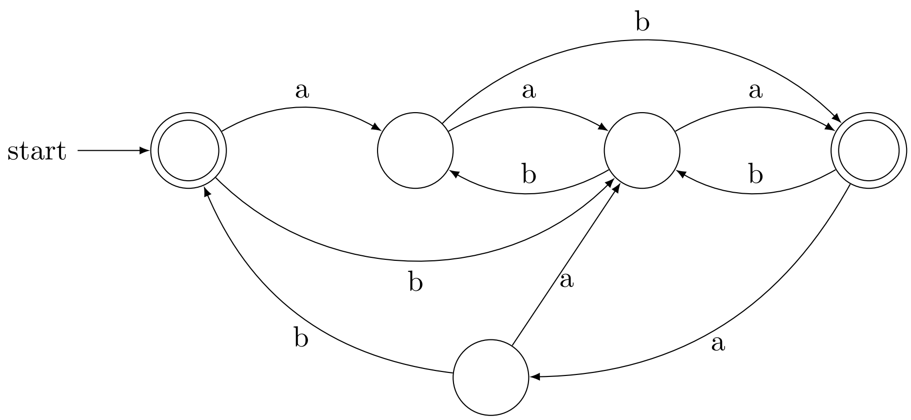
b) $L = \{x \in \{a,b\}^* :$ the number of a's in x is twice the number of b's in $x\}$

c) $L = \{x \oplus y \ominus z : x, y, z \in \{0,1\}^*$, bin($x$)+bin($y$) =bin($z$)$\}$ for $\Sigma = \{0, 1, \oplus, \ominus\}$. And bin($x$) for $x \in \{0,1\}^*$ is the number associated with x when interpreted as a binary number.

d) $L = \{x \in \{0,1\}^* : x$ ends in $101\}$

Problem 3: State Minimization (30/100)

Consider the following automaton. Find the minimal equivalent DFA.



Problem 4: Classes (40/100)

Let k be an integer, and define L_k to be the language of strings over $\{0, 1\}$ that have a 1 in the kth-to-last position. Prove that L_k is regular and that every DFA for L_k has exponentially many states with respect to k. Prove that L_k has an NFA with $O(k)$ states and a Regular Expression of length $O(k)$.