

Problem Set 10

This problem set is due on Friday, April 24, by 5pm. Please submit your solution online using bcourses, as a pdf file. You can type your solution, or handwrite it. If you handwrite it, then either scan it or take a good resolution picture of each page and then collate the pictures and export them to a *single* pdf file.

You can use the fact that $NL=coNL$ in your solutions.

Problem 1 (35/100)

Consider the problem ShortestPath = $\{(G, s, t, k) : \text{the shortest path from } s \text{ to } t \text{ in } G \text{ has length exactly } k\}$ where $G = (V, E)$ is a directed graph, s, t are vertices, and the length of a path is the number of edges in the path.

Show that ShortestPath is NL-complete.

Problem 2 (35/100)

Show that $SPACE(n)$ is not closed under polynomial-time reductions. That is, when $A \leq_m^p B$ for languages $B \in SPACE(n)$ and some other language A , it may not be the case that $A \in SPACE(n)$ as well.

[Note for contrast that P was closed under polynomial-time reductions since $A \leq_m^p B$ and $B \in P$, implies $A \in P$.]

Problem 3 (30/100)

Show that $NP \neq SPACE(n^2)$.