Problem Set 1

This problem set is due on Friday, January 30, by 5pm. Please submit your solution online using bcourses, as a pdf file.

You can type your solution, or handwrite it. If you handwrite it, then either scan it or take a good resolution picture of each page and then collate the pictures and export them to a *single* pdf file.

Problem 1: Repeat (15/100)

Let $\Sigma = \{a, b\}$. Let L be a language over Σ, consisting of strings of length at least 2, where the first letter is the same as the last letter, and the second letter is the same as the second to last letter. For example, $a \notin L$, $b \notin L$, $aa \in L$, $aaa \in L$, $aba \in L$, $bbaabba \notin L$.

Design a DFA that accepts L.

Problem 2: Multiples of 3 (25/100)

Let $\Sigma = \{0, 1\}$, and L be the language of all binary strings that are the binary representations of an integer multiple of 3 (leading zeroes are allowed). For example, $0 \in L$, $101 \notin L$, $1100 \in L$.

Design a DFA that accepts L

Problem 3: Prefix, Suffix, Mid (60/100)

Let Σ be an alphabet. Ex: $\Sigma = \{a, b, ..., z\}$.
Let $\circ$ be the concatenation operator. Ex: "hello" $\circ$ "world" = "helloworld"
Let L be a language over alphabet Σ, where prefix(L) $= \{x \mid \exists z : x \circ z \in L\}$, suffix(L) $= \{z \mid \exists x : x \circ z \in L\}$, and mid(L) $= \{y \mid \exists x, z : x \circ y \circ z \in L\}$.

- Given a DFA that accepts L, design a DFA or NFA that accepts prefix(L). (20 points)

- Given a DFA that accepts L, design a DFA or NFA that accepts suffix(L). (20 points)

- Given a DFA that accepts L, design a DFA or NFA that accepts mid(L). (20 points)