Problem Set 2

Due on Friday May 21, 2010

1. The LOGVC problem is the problem in which, given a graph $G = (V, E)$ we want to find a vertex cover C such that $\log_2 |C|$ is minimized.

   (That is, LOGVC is the same the minimum vertex cover, except that the cost function that we minimize is the logarithmic of the size of the set instead of the size of the set.)

   Prove that the LOGVC problem has a PTAS (see below for a definition of PTAS).

2. The LOGIS problem is the problem in which, given a graph $G = (V, E)$ we want to find an independent set S such that $\log_2 |S|$ is maximized.

   Prove that the LOGIS problem does not have a PTAS unless **P= NP**.

3. We proved that there are constants B and ϵ such that, given an instance ϕ of 3SAT such that every variable appears in at most B clauses it is NP hard to distinguish the cases: (i) ϕ is satisfiable from (ii) at most a $(1 - \epsilon)$ fraction of the clauses can be simultaneously satisfied.

   Prove that there is an ϵ' such that, given an instance ϕ of 3SAT such that every variable appears in at most 3 clauses, it is NP-hard to distinguish the cases: (i) ϕ is satisfiable from (ii) at most a $(1 - \epsilon')$ fraction of the clauses can be simultaneously satisfied.

Notes:

- A *Polynomial Time Approximation Scheme* (PTAS) for an optimization problem is an algorithm $A(\cdot, \cdot)$ that on input an instance I of the problem and a parameter ϵ outputs a solution whose cost is at least $(1 - \epsilon)$ times the optimum (for a maximization problem, or at most $(1 + \epsilon)$ times the optimum for a minimization problem) and such that, for every fixed $\epsilon > 0$, the running time of $A(I, \epsilon)$ is polynomial in the length of I. (This means that a running time of $2^{O(1/\epsilon)} \cdot |I|^2$, or $O(|I|^{O(1/\epsilon)})$ is allowed, but a running time of $\frac{1}{\epsilon} \cdot 2^{|I|}$ is not.)
- Although we did not prove it in class, for problem (2) you can use the fact that there is a constant $\epsilon > 0$ such that it is NP-hard to approximate the Maximum Independent Set within $1/n^{\epsilon}$.