Problem Set 2

This problem set is not due and it will not be graded. We will discuss it in the February 24 lecture.

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1. Let G be a graph sampled from the $G_{n,\frac{1}{2}}$ Erdös-Renyi random graph distribution, let A be the adjacency matrix of G, let D be the diagonal matrix of degree of the vertices of G and let $L := D - A$ be the non-normalized Laplacian matrix of G.

   - Use one of the matrix Chernoff bounds we stated in class to prove that, with probability that tends to 1 with n,

     $$||L - \mathbb{E}\, L|| \leq O(\sqrt{n \log n})$$

   - Prove that the above result is tight, that is, there are absolute constant $p >, c > 0$ such that

     $$\mathbb{P}[||L - \mathbb{E}\, L|| \geq c \cdot \sqrt{n \log n})] \geq p$$

   - We used an ϵ-net argument to argue that with high probability $||A - \mathbb{E}\, A|| \leq O(\sqrt{n})$. What goes wrong if we try to use the same argument on L?