

Problem Set 1

This problem set is not due and it will not be graded. We will discuss it in the February 17 lecture.

Although all results are provable in general graph, you may restrict yourself to regular graphs, if that restriction helps you.

1. Suppose that $G = (V, E)$ is an undirected graph such that the normalized Laplacian of G has largest eigenvalue 2 with multiplicity 2. Prove that this implies that G is disconnected.
2. An undirected graph $G = (V, E)$ is 3-colorable if it is possible to assign to each vertex a label chosen from the set $\{a, b, c\}$ in such a way that all edges have endpoints that are labeled differently. Prove that the largest eigenvalue of the normalized Laplacian of G is at least 1.5.
3. An independent set in an undirected graph $G = (V, E)$ is a set of edges $I \subseteq V$ such that there is no edge in the graph having both endpoints in I . Prove that if G has an independent set of size at least k , then the largest eigenvalue of the normalized Laplacian is at least $n/(n - k)$. Can you deduce the solution to problem (2) as a corollary of this result?