Problem Set 3

This problem set covers the material of the third quarter of the course: lectures 13-19. This problem set is not due and it will not be graded, but you are welcome to come to office hours to discuss your solutions.

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1. Suppose that we are given the use of two 3-D printers for T minutes each, and that we have n objects that we would like to print, to sell later. Object i would take t_i minutes to print, and would earn us v_i Euro in profit once sold. Devise an algorithm that runs in time polynomial in T and n and that, Given T, $t_1, \ldots, t_n$ and $v_1, \ldots, v_n$ decides a schedule of which print jobs to run on the first printer and which print jobs to run of the second printer, so that both printers can complete their assigned jobs in $\leq T$ minutes, and that the total profit is maximized (possibly, not all n objects will be printed).

   Note: these are one-of-a-kind object, so you should not print more than one copy of the same object, or else you will only be able to sell one of them.

2. Suppose that we redefine Edit Distance to allow one operation to delete an arbitrary sequence of consecutive letters, in addition to the operations of inserting one letter and of deleting one letter. For example, according to this definition, the Edit Distance between *artisan* and *cat* is 2.

   Describe and analyze an algorithm that, given two strings both of length $\leq n$, runs in time polynomial in n and computes the modified Edit Distance between them.

3. Say that two Boolean circuits C and C' are *equivalent* if, for every input they have the same output, that is
$$\forall x \;\; C(x) = C'(x)$$
   Show that if P=NP there is a polynomial time algorithm that, given a boolean circuit C, finds a smallest circuit C' such that C' is equivalent to C. By "smallest" we mean the circuit with the fewest gates.

4. We say that the input x_1 of a Boolean circuit $C(x_1, \ldots, x_n)$ is *influential* if there is a setting $a_2, \ldots, a_n$ for the other variables such that
$$C(0, a_2, a_3, \ldots, a_n) \neq C(1, a_2, a_3, \ldots, a_n)$$
   Prove that problem of deciding if the first input of a Boolean circuit is influential is NP-complete

   [Hint: reduce from Circuit SAT]